A CONCEPT WITHOUT REPRESENTATION IS STILL LIMP: 
A REFLECTION ON ELEMENTARY SCHOOL 
MATHEMATICS LEARNING ON INTERGER 
MULTIPLICATION OPERATION SUBJECT MATTER

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ABSTRACT
Mathematics is a subject that requires students to learn how to reason properly. Indeed, Mathematics in it contains concepts that are abstract in nature, but the abstract concepts can be bridged by an expression called representation. The most important thing in learning Mathematics is the process how students think, even though a process will always lead to a result. Therefore, it is important for teachers of Mathematics, especially in elementary schools, to use various forms of representation to present Mathematical concepts to be more alive and to be recognized by students to be able to be brought home to them and use correct Mathematical concepts in their daily lives. In this study an example is given of the representation of the integer multiplication operation in elementary school Mathematics learning.

Keywords: Mathematics concepts, integer multiplication, Mathematics representation

1. INTRODUCTION
Mathematics is a subject which actually has very broad benefits for the development of science and technology. However, not all understand and feel the benefits immediately how math made an application in everyday life. In fact, not least among which considers that Mathematics can only be learnt at school. This shows that the subjects of Mathematics was considered not a meaningful lesson although the meaning and the benefits are enormous.

Why there is still the assumption that Math lessons are less meaningful is not due to one or two factors only, but there are a variety of factors in it. Not a few research stated that the problem of learning math in school only focuses on the method of teaching by the teacher. It is true with learning methods and/or a fun technique will cause the students learn the concepts well. The requirement here is that the teacher must be mastered the material and Mathematical competence-competence.

In addition to the methods that are fun, there are things that makes understanding math concepts students attached with robust, i.e. with the use of appropriate representation. To that end, every learning Mathematics shall associate the concept with an appropriate representation. When students understand the concept only by Mathematical calculations, and less understanding of appropriate representation, then there is the possibility of students only understand concepts in school only.

For example, when students are asked to distinguish the operation of $2 \times 3 \times 3$ with 2, maybe not a few students, even adults, will say the same multiplication operations both with the same results even though the number of different layout. The answers thus turn out to
be less precise, according to people who have already mastered the science of Mathematics.

Another example is when children are asked to determine the number of cows feet if there are 3 cows. The question is how the writing of the true multiplication operations according to the definition of multiplication operations? If the teacher still merrrepresentasikannya with reverse operation, meaning that teachers haven’t been able to usher students towards an understanding of the true concept, although in computing the right calculations produce.

The illustration shows that the representation be things that support the creation of Mathematical concepts. According to the National Research Council (2001), Mathematics require representation. The reason is because the abstract Mathematical nature, people have access to enter the Mathematical idea with only through the representation of the idea. Mathematical representation refers to the process or product in learning (NCTM, 2000). The term for the process and the product is that it can be monitored externally or happened “internally”, in the minds of people who do the math.

Mathematics has always menitikberakan on the use of reason, or in the language of the lighter is thinking. Thought is a mental process to bear fruit of thought. This means that learning math should stimulate the children follow the learning not only in situations that are fun, but facilitate the children to be able to bear the thought. Thus, teachers can guide students through appropriate representation to assist the formation of Mathematical concepts, because according to Kalathil & Sherin (2000, Kartini, in 2009), the representation is used to provide information to teachers about How do students think about a Mathematical idea or context.

In this paper will be discussed more deeply about: 1) the Mathematical concepts of Mathematical representation, 2), 3) relation between Mathematical concepts and Mathematical representation and 4) example Mathematical representation of multiplication operations on whole numbers primary school.

2. DISCUSSION
2.1 Mathematics Concepts

According to Sadiq and Efficacious (2011), a concept is an abstract idea that allows someone to classify an object and explain whether the object is or is not an example. According to Gagne (1975), a concept is one of the intellectual skills. Intellectual skills include the ability to “find out how” information, not “knowing” him. Further, Gagne (1975) distinguish the concept into two, namely concrete concepts and concept definition. Concrete concept is a single object or event by showing or giving names. Whereas the definition concept is the concept of an object that cannot be classified with the “show it”, but rather should use sentence (or statements). A definition is used to identify a particular instance of the concept of a group of objects.

The concrete concept can be replaced or given extra meaning by definition, concept and often the main purpose is such a topic of education. The concept of “two” and “three” was a concrete concept is for children, which can be easily recognized (by pointing) without the use of a definition. In learning in school, this concept can be replaced with a definition of the concept, where “three” is defined “a group consisting of combined/summation between one of two”. Definition concept is actually a classification rule.

According to Gagne (1975), a student can be said to have mastered the definition concept when he can demonstrate, or show how to use a definition, with classifying examples of the concept. It should be noted that the student does not need to declare the definition in order to demonstrate that he knew the concept of it. According to Hadi (2017), the process of learning math should be emphasized in the concept known students. Each student has a set
of knowledge that has been perceived as a result of interaction with the environment or learning process before. Therefore, the point of departure of the learning sequence should give a real experience for the students so that they can be directly involved in personal activity in Mathematics.

According to Karso (2014), math concepts are grouped into three types: basic concepts, concepts that develop, and concepts that should nurture the skills. The basic concept is a concept that is first studied by students from a number of concepts that are given. Therefore, after this basic concept is embedded then it will be a prerequisite in understanding the concept. The concept is developed from the basic concept is the nature or the implementation of the basic concepts. Meanwhile the concept of the third type need to get attention and coaching of teachers so that students have the skills in using or displaying the basic concepts or concepts that develop.

For instance, in learning grade 1 elementary Math, the basic concept is “Make two numbers with one number with results of up to 5”. The concept developed from that basic concept is “know the nature of the exchanges on the sums” and “determine the number of couples whose number is known, and no more than 5”. Meanwhile, the concept of the third type is “completing the simple stories”.

2.2. Mathematics Representation

NCTM (2000) states that the ways in which ideas of Mathematics are represented are fundamental to how people can understand and use these ideas. The term representation refers both to the process as well as to the product — in other words, to the act of capturing the Mathematical concepts or relationships in some forms and the form itself. In addition, the term representation applies to processes and products which can be monitored externally or happened “internally”, in the minds of people who do the math.

Based on the theory of stages of learning from Jerome Bruner (in Reys, et al., 2009), representational thinking is based on the use of images or other forms of representation. Children involved with pictorial and/or oral information based on the real world. Some form of representation — such as diagrams, graphics display, and symbolic expression — has long been a part of school Mathematics (NCTM, 2000). Moreover, Lesh, et al. (2003, in Van de Walle, 2013) distinguishes 5 types of representations to demonstrate understanding for any topics, i.e. a real-world situation, a model of manipulative, symbol of written, oral or verbal, language and images.

When students get access to Mathematical representation and ideas that they represent, they have a set of tools that significantly expand their capacity to think Mathematically. Representation should be treated as an important element in supporting student understanding of Mathematical concepts and relationships; in communicating the Mathematical approach, arguments, and an understanding of self and others; in recognizing the connection between math concepts are related; and in applying Mathematics to a realistic problem situations through modeling (NCTM, 2000).

The National Research Council (2001) provides guidelines for assessing whether a representation can be used to communicate Mathematical ideas, as follows.

a. Transparency: how easy the Mathematical ideas can be seen through this representation?
b. Efficiency: does this representation support communication and effective use?
c. Generality: does the representation apply to a broad class of objects?
d. Clarity: is this representation not ambiguous and easy to use?
e. Precision: how close is this representation to the right value?
2.3 The Relationship between Mathematics Concepts and Mathematical Representation

The nature of abstract Mathematics makes many people think that Mathematics is a difficult subject. According to Hadi (2017), the eyes of learning Mathematics has become a scary specter for the majority of school children. It is very close to math learning result which is always bad. Therefore, according to Kartini (2009), representation is indispensable in understanding the Mathematical concepts or problem solving. In addition, representation needed to add Mathematical ideas (National Research Council, 2001).

According to the NCTM (2000), moving from one representation to another is an important way to add in-depth understanding of the newly-formed ideas. If children can use a lot of representation, it is important that they can translate between the representations (National Research Council, 2001). Strengthening the ability to move between these representations improves the understanding and retention of students (Van de Walle, et al., 2013).

Based on the theory of Bruner (in Van de Walle, et al., 2013), there are three stages of learning, i.e. the stages of enactive, iconic, and symbolic. In Mathematics, these stages are better known as concrete, semi-concrete (representational), and abstract. This model reflects the order in which the focus moves from instructional on concrete representation (manipulative material) and the model into a semi-concrete representation (images or image) and the image to abstraction (using only numbers or mental problem solving). Normally students will develop starting from the use of concrete objects to explore new concepts. When students share their thoughts that show they are beginning to understand the Mathematical concepts, there could be a shift to a semi-concrete or semi-abstract representation.

When you think about how learning should move from the concrete to the abstract, it is important to remember that the concrete is a relative term (Reys, et al., 2009). Symbols and formal representations of Mathematical ideas follow naturally from the concrete level, but only after the conceptualization and understanding of meaning has been set. Without such an understanding, the kids do not feel comfortable working with the symbols of Mathematics, and Mathematics does not make sense for them. As noted in the framework of Bruner, children need the opportunity to work with objects in the physical world before they are ready to work with images and other representations (Reys, et al., 2009). Then, after working with images and other representations, they will be ready to work with symbols.

Reys et al. (2009) states that children need a lot of experience with concrete models before they can work in a meaningful way with abstract symbols. Prior experience with the model will help children develop conceptual knowledge about a topic, such as able to talk about the topic and see the pictorial representation. In the end, the teacher can show children how a symbol can be used to record what has been expressed orally on concrete model or drawing.

2.4 An Example of a Mathematical Representation of Learning Multiplication Operations on Primary School Integer

Integer is the basic material of math in elementary school. Sonnabend (2010) argues that whole numbers helps us find the way and home, and also in tracking how many objects we have. But not a few of students as well as adults who do not know there is a difference between numbers in quantity terms and numbers in symbol terms. According to Hadi (2017:125), the numbers in symbol terms is the symbol of a numbers (in quantity terms) and are abstract. According to Japa and Suarjana
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(2012:100), “numbers (in quantity terms) is the abstract ideas to state the number of members of a set, while the symbol of number is the symbol written to mark some number.

Hadi (2017:125) presents a question, “what happens when elementary school students on low grade have to imagine the operation count represented by numbers in symbol terms?” Abstract Mathematics makes elementary school students have trouble because it has not been in abstract or formal thinking. Therefore it needs to be taught in accordance with the stage of their cognitive development (Jean Piaget’s theory), namely concrete operational stage.

On a concrete stage, students should look, feel, and move objects. For example, when students are doing the process of counting. The teacher will probably help students use the help of fingers or other objects and associate numbers with fingers or other objects (Hadi, 2017). For example, as described by Jarmita (2015), at the concrete stage, 2 blocks and 3 blocks are used to learn that they amount to 5 blocks. On the representational stage, an image can represent a real object. As an example; 0000 + 000 = 7. In the abstract stage, the numbers eventually replace the picture or graphic symbol, for example: 4 + 3 = 7.

But for the operation of multiplication is somewhat different. Multiplication is the operation of numbers which competencies are introduced to the students of class two second half (Mustaqin, 2017). Such competencies are mandated in the standard contents of the elementary school/Madrasah Ibtidaiyah about Standard Competency that is “doing multiplication and division of numbers up to 2 digits” and the Basic Competence says “do the multiplication of numbers which the result is the number two figure” (Depdiknas, 2006).

According to Sonnabend (2010), the definition of multiplication operation is for any whole numbers $a$ and $b$ with $a 
eq 0$, then $a \times b = b + b + \ldots + b$ as many $a$ times. In other words, the multiplication is repeated addition operation form. Sonnabend (2010) also classifies that there are 5 types of applications to whole numbers multiplication operation, namely (1) the same set, (2) the same size, (3) ranks (sets), (4) total area (size), (5) pairing. According to the author, efficient application that corresponds to the stages of development of students is the kind of (1), (2) and (3). Application (4) is suitable used when students are learning basic material about the area.

For example, to demonstrate the operation of $3 \times 4$. With the applications (1), (2) and (3), an example of the representation in a row is like in Figure 2.1 below. With the existence of examples like this, it is expected that teachers are no longer confused when describing the concept of multiplication. So that is not the case of the possibility of confusion about this concept, as in Figure 2.2 below.

![Figure 2.1 Representations of $3 \times 4$ that is correct and suitable for elementary students](image-url)
Furthermore, it should be observed if Math teachers want to make math story problem as simple as in the Introduction above, i.e., determine the number of cows feet if there are 3 cows. It is not enough just to determine that the result is 12 feet, but need to find how the children is facilitated as many as 12 feet of 3 cows.

3. CONCLUDING REMARKS

When teachers explain the concept of Mathematics, teacher should better understand in advance what is meant by the concept of it. After understanding the concept, teachers should not stop only at calculation of normal or only on the determination of the solution of the question of the story? The math is not quite simply by instilling the basic concept and the concept that develops, the concept should continue to be built on his skills, including skills in determining representation where appropriate, not only determine the outcome or solution, but how students find the concept with process thinking and pour it in a representation. Therefore, representation is essential to bridge concrete concept headed to concepts that are abstract. Without representation, the math does not become meaningful. Without representation, the math will become limp.

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